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Squeezing properties of the Kerr-down conversion system

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Abstract. In this paper we describe a new two-mode system, which consists of Kerr-like medium and down conversion process, called the Kerr-down conversion system. Under a certain condition we can obtain an exact solution of the dynamical equations of motion. For this system we investigate different kinds of quadrature squeezing, e.g., single-mode, two-mode and sum-squeezing. Also we give a more general definition of the principal squeezing. We show that the amounts of nonclassical effects produced by the Kerr-like and down-conversion processes separately are greater than those obtained from the Kerr-down conversion system where both the processes are in competition.

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1 Introduction

There are three important processes in nonlinear optics, namely, up-conversion, down-conversion and Kerr-like process. These processes took most of attention in the field of quantum optics. The up-conversion process cannot generate nonclassical effects, however, it switches energy between modes, e.g., signal and idler [1]. In this regard the up-conversion process is mainly used in the directional couplers to transfer data between waveguides [2]. The down-conversion, which is usually named in the literature as parametric amplifier, can generate nonclassical effects. Precisely, the degenerate and nondegenerate parametric amplifiers perfectly generate single-mode [3] and two-mode [4] squeezed states, respectively. Actually, squeezed states have been applied in the quantum information [5]. The single-mode state obtained from the nondegenerate parametric amplifier is 'super-classical' in the sense that the evolution of the system broadens the single-mode P distribution as a result of the spontaneous pump photon decays [6]. This behavior was named later self-decoherence [7]. Moreover, the parametric amplifier has been used in the observation of the interference effects. For instance, the fourth-order interference effects arise when pairs of photons produced in parametric amplifier are injected into Michelson interferometers [8]. Additionally, the second-order interference is observed in the superposition of signal photons from two coherently pumped parametric amplifiers when the paths of the idler photons are aligned [9]. Now we draw the attention to the interaction of light with the Kerr-like medium. This type of interaction is representative by generating cat states, namely, the Yurke-Stoler state [10]. The Yurke-Stoler state can generate nonclassical squeezing in spite of the fact that the photon-number distribution is Poissonian. The Kerr-like medium has been intensively studied for various kinds of the initial states aiming to obtain nonclassical squeezed light, e.g., [11]. Such studies are encouraged by the possible observation of the large values of the third-order optical nonlinearities in, e.g., the organic polymers [12].

The competition between the down-conversion (the Kerr-like medium) and up-conversion processes is of interest from the theoretical and experimental points of views, e.g., in the three-mode interaction [13] and in the nonlinear directional couplers [14,15]. Nevertheless, the competition between the Kerr-medium and nondegenerate parametric amplifier — as far as we know — has not been treated yet. Thus in this paper we investigate this system, i.e., the Kerr-down conversion system. For this system the exact solution can only be obtained under certain condition, as we shall see. As we mentioned above the Kerr medium and the down-conversion are important processes, so that the connection between them is important, too. Moreover, this system enables us to investigate the influence of the Kerr medium on the down-conversion and vice versa. For this system we investigate different types of quadrature squeezing such as single-mode, two-mode and sum squeezing. Also we develop the notion of the principal squeezing [16] for any type of the quadrature squeezing. It is worth reminding that the squeezed light can be measured in the homodyne detector where

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the signal is superimposed on the strong coherent beam of the local oscillator. Generally, we show that the nonclassical effects generated by this system are smaller than those obtained from the individual processes, i.e., each process destroys the nonclassical effects generated by the other one. This contradicts with what one could possibly expect, i.e., that the combination of these processes will increase the nonclassical effects. We treat this problem in the following order: in Section 2 we give the Hamiltonian model of the system and the basic equations and relations. In Section 3 we discuss the corresponding results.

2 Basic relations and equations

In this section we give the Hamiltonian model and solve the associated equations of motion. Also we write down the definitions of the quadrature squeezing and develop general definition to the principal squeezing. The Hamiltonian model for the Kerr-down conversion system can be expressed as

$$\frac{\hat{H}(t)}{\hbar} = \sum_{j=1}^{2}\left[\omega_j \hat{a}_j^\dagger \hat{a}_j + \chi_j \hat{a}_j^{\dagger 2}\hat{a}_j^2\right] + \overline{\chi}\hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2 - ik\left[\hat{a}_1\hat{a}_2\exp(i\omega t) - \hat{a}_1^\dagger \hat{a}_2^\dagger \exp(-i\omega t)\right], \quad (1)$$

where the waves are described by annihilation operators $\hat{a}_j$ and by frequencies ω_j $(j = 1, 2)$ of the first and second mode, respectively. The coupling constants χ_j and $\overline{\chi}$ are proportional to the third-order susceptibility $\chi^{(3)}$ and are responsible correspondingly for the self-action and cross-action processes of the modes. The coupling constant k is real and $\omega = \omega_1+\omega_2$. The Hamiltonian (1) can be obtained in terms of two modes which are interacting via a multi-layer nonlinear crystal comprising from the Kerr medium and down conversion medium. This could be possible with respect to the progress of preparation of new nonlinear crystals and improved laser sources [12,17].

The exact solution for the equations of motion for (1) can be obtained under the assumption that $\overline{\chi} = -2\chi_1 = -2\chi_2$. In this case $\hat{N} = \hat{n}_1 - \hat{n}_2$, where $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$, is a constant of motion and the solution takes the form:

$$\hat{A}_1(t) = \exp(-2i\overline{\chi}\hat{N}t)\left\{\hat{a}_1(0)C + \hat{a}_2^\dagger(0)S\right\},$$
$$\hat{A}_2(t) = \exp(2i\overline{\chi}\hat{N}t)\left\{\hat{a}_2(0)C + \hat{a}_1^\dagger(0)S\right\}, \quad (2)$$

where $\hat{A}_j(t) = \exp(i\omega_j t)\hat{a}_j$ and we have used the abbreviations

$$C = \cosh(kt), \quad S = \sinh(kt). \quad (3)$$

It is evident that the expressions (2) include the amplification and periodical features of the down-conversion and Kerr-like processes, respectively. The Kerr process is represented by the non-trivial phase factor, which plays an essential role in occurrence of the nonclassical effects.

As we investigate various types of squeezing, i.e. single-mode, two-mode, and sum squeezing, we define two general quadratures as $\hat{X} = (\hat{B}+\hat{B}^\dagger)/2$ and $\hat{Y} = (\hat{B}-\hat{B}^\dagger)/2i$, which satisfy the commutation rule $[\hat{X}, \hat{Y}] = \hat{D}/2i$. The operators $\hat{B}$ and $\hat{D}$ will be specified in the text. Squeezing factors associated with the $\hat{X}$ and $\hat{Y}$ can be expressed, respectively, as

$$F = \frac{1}{|\langle\hat{D}\rangle|}\left[2\mathrm{Re}\langle\hat{B}^2\rangle + 2\langle\hat{B}^\dagger\hat{B}\rangle + \langle\hat{D}\rangle - |\langle\hat{D}\rangle| - 4[\mathrm{Re}\langle\hat{B}\rangle]^2\right]$$
$$G = \frac{1}{|\langle\hat{D}\rangle|}\left[2\langle\hat{B}^\dagger\hat{B}\rangle - 2\mathrm{Re}\langle\hat{B}^2\rangle + \langle\hat{D}\rangle - |\langle\hat{D}\rangle| - 4[\mathrm{Im}\langle\hat{B}\rangle]^2\right]. \quad (4)$$

We conclude this section by shedding light on the principal squeezing [16]. For principal squeezing the quadrature is defined as:

$$\hat{X}_\phi = \frac{1}{2}[\hat{B}\exp(-i\phi) + \hat{B}^\dagger\exp(i\phi)]. \quad (5)$$

The value of the angle ϕ can be controlled by the homodyne detector to obtain the maximum amount of squeezing. The squeezing factor related to (5) is

$$V_\phi = \frac{1}{|\langle\hat{D}\rangle|}\left[4\langle(\triangle\hat{X}_\phi)^2\rangle - |\langle\hat{D}\rangle|\right]. \quad (6)$$

By evaluating the extreme values for (6) with respect to ϕ, one can obtain general form for the principal squeezing as

$$V = \frac{1}{|\langle\hat{D}\rangle|}\left[\langle\hat{D}\rangle + 2\langle\hat{B}^\dagger\hat{B}\rangle - (2\langle\hat{B}\rangle\langle\hat{B}^\dagger\rangle + |\langle\hat{D}\rangle|) - 2\left|\langle\hat{B}^2\rangle - \langle\hat{B}\rangle^2\right|\right]. \quad (7)$$

The definition (7) is more general than that given in [16] in which the principal squeezing definition is derived only for the single-mode and two-mode cases through lengthy calculations.

3 Discussion of the results

We use the results given in the previous section to investigate the quadrature squeezing for the system under consideration. We focus the attention on the single-mode, two-mode and sum squeezing. We assume that the two modes are initially prepared in the coherent states $|\alpha_1, \alpha_2\rangle$ with real amplitudes α_j. Also we shed light on the evolution of the principal squeezing for each type. This will be done in the following.

3.1 Single-mode squeezing

As is well-known that the Kerr-like medium can provide nonclassical squeezing [11], however, the nondegenerate parametric amplifier cannot generate single-mode squeezing as a result of the self-decoherence [7]. In the present system we found that the Kerr-like medium is responsible for generating periodical single-mode squeezing provided

$$
\begin{aligned}
F_1^{(1)}(t) &= 2[\alpha_1^2 C^2 + 2\alpha_1\alpha_2 SC + S^2(\alpha_2^2+1)] + 2\left[\alpha_1^2 C^2\cos\Theta_+ + \alpha_2^2 S^2 \sin\theta + 2\alpha_1\alpha_2 CS\cos\Theta_-\right]\exp[\epsilon_1\sin^2(2\chi t)] \\
&\quad - 4\left[\alpha_1 C\cos(\epsilon_2\sin(2\chi t)) + \alpha_2 S\cos(2\chi t - \epsilon_2\sin(2\chi t))\right]^2 \exp[\epsilon_1 \sin^2(\chi t)], \\
G_1^{(1)}(t) &= 2[\alpha_1^2 C^2 + 2\alpha_1\alpha_2 SC + S^2(\alpha_2^2+1)] - 2\left[\alpha_1^2 C^2\cos\Theta_+ + \alpha_2^2 S^2 \sin\theta + 2\alpha_1\alpha_2 CS\cos\Theta_-\right]\exp[\epsilon_1\sin^2(2\chi t)] \\
&\quad - 4\left[\alpha_1 C\sin(\epsilon_2\sin(2\chi t)) - \alpha_2 S\sin(2\chi t - \epsilon_2\sin(2\chi t))\right]^2 \exp[\epsilon_1 \sin^2(\chi t)], \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
F_2(t) &= \frac{1}{2}\left[F_1^{(1)}(t) + F_1^{(2)}(t)\right] + 4\left[\alpha_1\alpha_2\left(S^2+C^2\right) + SC\left(\alpha_1^2+\alpha_2^2+1\right)\right]\cos(2\chi t) \\
&+ 4\Big\{\alpha_1\alpha_2\left(C^2+S^2\right)\cos(\epsilon_2\sin(4\chi t)) + CS\alpha_1^2\cos(4\chi t + \epsilon_2\sin(4\chi t)) + CS\alpha_2^2\cos(4\chi t - \epsilon_2\sin(4\chi t))\Big\}\exp\left[\epsilon_1\sin^2(2\chi t)\right] \\
&- 8\left[\alpha_1 C\cos(\epsilon_2\sin(2\chi t)) + \alpha_2 S\cos(2\chi t - \epsilon_2\sin(2\chi t))\right]\left[\alpha_2 C\cos(\epsilon_2\sin(2\chi t)) + \alpha_1 S\cos(2\chi t - \epsilon_2\sin(2\chi t))\right] \\
&\times \exp\left[2\epsilon_1\sin^2(\chi t)\right],
\end{aligned}
$$

$$
\begin{aligned}
G_2(t) &= \frac{1}{2}\left[G_1^{(1)}(t) + G_1^{(2)}(t)\right] - 4\left[\alpha_1\alpha_2(S^2+C^2) + SC(\alpha_1^2+\alpha_2^2+1)\right]\cos(2\chi t) \\
&+ 4\Big\{\alpha_1\alpha_2\left(C^2+S^2\right)\cos(\epsilon_2\sin(4\chi t)) + CS\alpha_1^2\cos(4\chi t + \epsilon_2\sin(4\chi t)) + CS\alpha_2^2\cos(4\chi t - \epsilon_2\sin(4\chi t))\Big\}\exp\left[\epsilon_1\sin^2(2\chi t)\right] \\
&- 8\left[\alpha_1 C\sin(\epsilon_2\sin(2\chi t)) - \alpha_2 S\sin(2\chi t - \epsilon_2\sin(2\chi t))\right]\left[\alpha_2 C\sin(\epsilon_2\sin(2\chi t)) - \alpha_1 S\sin(2\chi t - \epsilon_2\sin(2\chi t))\right] \\
&\times \exp\left[2\epsilon_1\sin^2(\chi t)\right], \qquad (11)
\end{aligned}
$$

that k is very small. This means that the competition between the two processes, i.e. Kerr-like medium and down-conversion, may destroy the nonclassical effects inherited in the individual ones. We show this fact for the first mode. In this case we have $\hat{B}(t) = \hat{A}_1(t), \hat{D} = 1$ and the squeezing factors have the forms:

see equations (8) above

where the subscript (superscript) 1 (1) means that these quantities are related to the single-mode case (first mode). Also in (8) we have used the following abbreviations:

$$
\begin{aligned}
\epsilon_1 &= -2(\alpha_1^2+\alpha_2^2), \qquad \epsilon_2 = \alpha_1^2 - \alpha_2^2, \\
\Theta_\pm &= 2\chi t \pm \epsilon\sin(4\chi t), \quad \theta = 6\chi t - \epsilon\sin(4\chi t). \qquad (9)
\end{aligned}
$$

One can easily check that when $(k, \alpha_2) = (0, 0)$ the relations (8) reduce to those of the single-mode Kerr-like medium. Now let us restrict our discussions to the case in which $\alpha_1 = \alpha_2$. From (8) and (9) one can easily realized that maximum squeezing occurs when $\chi t = m\pi/2$, where m is a positive integer. This means that values of the χ are responsible for the periodicity, i.e. the degree of harmonics, of the squeezing. For these values of the interaction times the expressions (8) reduce to

$$
\begin{aligned}
F_1^{(1)}(t) &= 2S^2 - 4\alpha_1^2\exp(\epsilon_1 - 2kt), \\
G_1^{(1)}(t) &= 4\alpha_1^2(C+S)^2 + 2S^2. \qquad (10)
\end{aligned}
$$

From (10) it is evident that squeezing occurs only in the x -quadrature when k is very small or zero. This indicates that the amplification of the down-conversion (related to increase of quantum noise) decreases the non-classicality produced by the Kerr-like medium. In Figure 1 we have plotted the principal squeezing and the quadrature squeezing for the first mode when $(\chi, k) = (0.5, 0)$. It is obvious that the principal squeezing provides amount of nonclassical squeezing greater than that of the quadrature squeezing (compare the solid and long-dashed curves as well as the short-dashed and the star-centered curves). The comparison between the long-dashed and star-centered curves shows that the entanglement between modes decreases the nonclassical squeezing. Finally, we have noted that the higher the values of α_2 the smaller the values of the squeezing.

3.2 Two-mode squeezing

We study here the two-mode squeezing in which the correlation between modes starts to play a role. As we mentioned in the Introduction that the nondegenerate parametric amplifier produces perfect nonclassical squeezing in one of the two-mode quadratures. Furthermore, the Kerr-like medium can produce nonclassical two-mode squeezing, too. Thus the overall behavior of the system could be expected to increase the amount of squeezing. Nevertheless, we have noted that this is not so. This will be seen as follows: for the two-mode squeezing $\hat{B}(t) = \hat{A}_1(t) + \hat{A}_2(t), \hat{D} = 2$ and the squeezing factors can be evaluated thus:

see equations (11) above

where $G_1^{(j)}(t)$ and $F_1^{(j)}(t)$ are the jth-single-mode squeezing factors. We start the discussion by investigating the influence of the entanglement on the two-mode Kerr-like

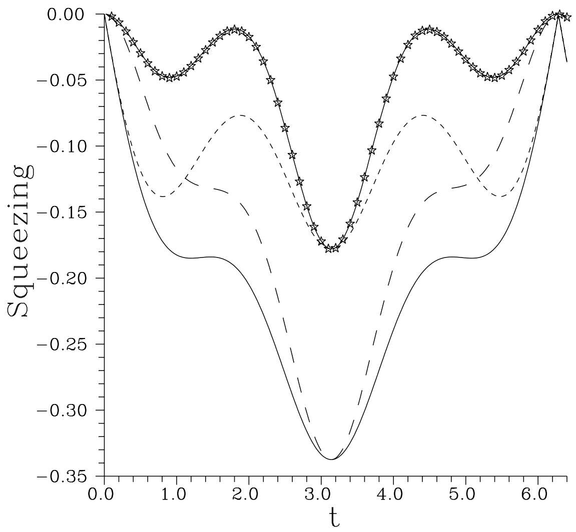

Fig. 1. The principal squeezing and the squeezing factor $F_1^{(1)}(t)$ for the first mode against the interaction time t when $(\chi, k) = (0.5, 0)$ and $(\alpha_1, \alpha_2) = (0.4, 0)$ (solid and long-dashed curves) and $(0.4, 0.4)$ (short-dashed and star-centered curves).

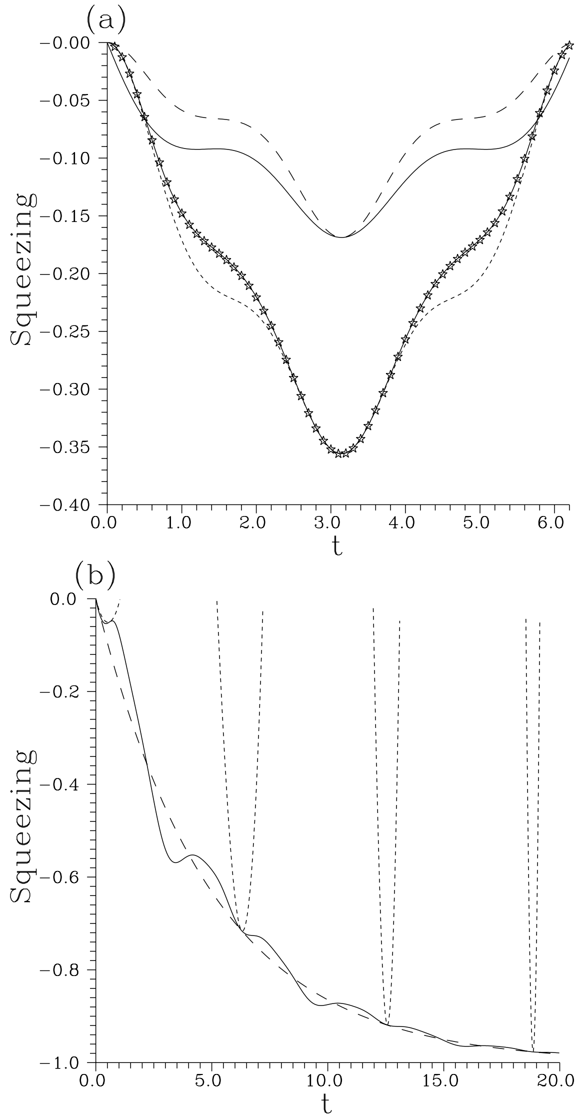

Fig. 2. The principal squeezing and squeezing factor for the two-mode case against the interaction time t. (a) For the same situation as in Figure 1. (b) Principal squeezing and $G_2(t)$ when $(\alpha_1, \alpha_2) = (0.4, 0)$ and $(\chi, k) = (0.5, 0.1)$ (solid and short-dashed curves). The long-dashed curve in (b) is given for $(\chi, k) = (0, 0.1)$.

medium. Thus we set $k = 0$ in (3). In this case the maximum amount of squeezing can periodically occur when $\chi t = m\pi$. At these values of the interaction time the expressions (3) reduce to

$$F_2(t) = \frac{1}{2}\left[F_1^{(1)}(t) + F_1^{(2)}(t)\right],$$
$$G_2(t) = \frac{1}{2}\left[G_1^{(1)}(t) + G_1^{(2)}(t)\right]. \qquad (12)$$

From (12) and the information given in Section 3.1 it is evident that when $\alpha_1 \neq 0$ and $\alpha_2 = 0$, $F_1^{(2)}(t) = 0$ and $F_1^{(1)}(t)$ can provide squeezing. Nevertheless, when $\alpha_j \neq 0$ both of the $F_1^{(1)}(t)$ and $F_1^{(2)}(t)$ provide nonclassical squeezing. This indicates that the amount of squeezing produced by the latter case is greater than that of the former case. This fact is remarkable in Figure 2a for given values of the interaction parameters. Moreover, the comparison between Figure 1 and Figure 2a shows that for certain values of the interaction time the nonclassical effects produced by the two-mode squeezing are greater than those of the single-mode squeezing. This manifests the role of the correlation between modes. Now we draw the attention to the general case, which is plotted in Figure 2b. From this figure one can observe that when $\chi = 0$ the nonclassical squeezing occurs in the y-quadrature only (see the long-dashed curve), which is rapidly increasing as the interaction time evolves. From the short-dashed curve in Figure 2b one can observe that the Kerr-like medium causes the nonclassical squeezing occurring periodically in the y-quadrature only with maximum values as those of the nondegenerate parametric amplifier. This indicates in the system under consideration that the kerr-like medium decreases the non-classicality produced by the down-conversion process. Finally, the comparison between the different curves in Figure 2b shows that the principal squeezing produces pure nonclassical effects which for particular values of the interaction time are greater than those obtained from the quadrature squeezing.

3.3 Sum squeezing

It is worth referring that sum squeezing has been calculated in nonlinear optics for four-wave sum [18] and difference [19] frequency generation. In this part we shall investigate sum squeezing for the present Hamiltonian model. In this case we have $\hat{B}(t) = \hat{A}_1(t)\hat{A}_2(t)$ and $\hat{D}(t) = \hat{A}_1^\dagger(t)\hat{A}_1(t) + \hat{A}_2^\dagger(t)\hat{A}_2(t)$. Now let us start the

$$F(t) = \frac{2\left\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\hat{A}_2^\dagger(t)\hat{A}_2(t)\right\rangle + 2\left[\mathrm{Re}\left\langle \hat{A}_1^{\dagger 2}(t)\hat{A}_2^2(t)\right\rangle_{\chi=0}\right]\cos(4\chi t) - 4\left[\mathrm{Re}\left\langle \hat{A}_1^\dagger(t)\hat{A}_2(t)\right\rangle_{\chi=0}\right]^2\cos^2(2\chi t)}{\left\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\right\rangle + \left\langle \hat{A}_2^\dagger(t)\hat{A}_2(t)\right\rangle},$$

$$G(t) = \frac{2\left\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\hat{A}_2^\dagger(t)\hat{A}_2(t)\right\rangle - 2\left[\mathrm{Re}\left\langle \hat{A}_1^{\dagger 2}(t)\hat{A}_2^2(t)\right\rangle_{\chi=0}\right]\cos(4\chi t) - 4\left[\mathrm{Im}\left\langle \hat{A}_1^\dagger(t)\hat{A}_2(t)\right\rangle_{\chi=0}\right]^2\sin^2(2\chi t)}{\left\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\right\rangle + \left\langle \hat{A}_2^\dagger(t)\hat{A}_2(t)\right\rangle}, \quad (15)$$

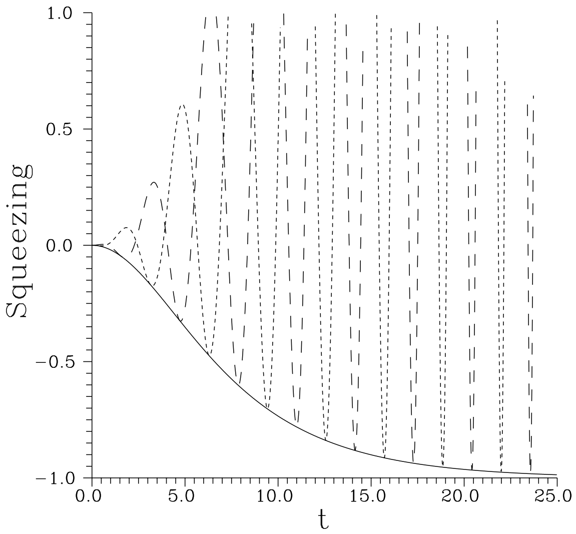

Fig. 3. Sum squeezing against the interaction time t for $(\alpha_1, \alpha_2, k) = (0.4, 0, 0.1)$ and when $\chi = 0$ (solid curve-y-quadrature), 0.5 (short-dashed and long-dashed curves for y- and x-quadratures, respectively). Also the solid curve represents the principal squeezing for the case $\chi = 0.5$.

discussion with the case $k = 0$, i.e., interaction between two modes via Kerr-like medium. Thus we obtain

$$F(t) = G(t) = 0. \quad (13)$$

This means that we have minimum uncertainty sum-squeezing, i.e. the two-mode-Kerr system cannot generate nonclassical squeezing. On the other hand, when $\chi = 0$ the system can exhibit squeezing only in the y-quadrature and the associated squeezing factor takes the form

$$G(t) = \frac{2\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\hat{A}_2^\dagger(t)\hat{A}_2(t)\rangle - 2\mathrm{Re}\langle \hat{A}_1^{\dagger 2}(t)\hat{A}_2^2(t)\rangle_{\chi=0}}{\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\rangle + \langle \hat{A}_2^\dagger(t)\hat{A}_2(t)\rangle}$$
$$= \frac{-2(\alpha_1^2 + \alpha_2^2 + 1)S^2C^2 - 4\alpha_1\alpha_2 SC}{\langle \hat{A}_1^\dagger(t)\hat{A}_1(t)\rangle + \langle \hat{A}_2^\dagger(t)\hat{A}_2(t)\rangle}. \quad (14)$$

From (14) it is evident that the nonclassical squeezing is monotonically increasing as the interaction time evolves. This is obvious from the solid curve in Figure 3 for given values of the interaction parameters. Now we draw the attention to the general case in which $\chi \neq 0$ and $k \neq 0$. The squeezing factors for this case take the forms

see equations (15) above

where Re and Im stand for the real and imaginary parts. From (14) and (15) one can realize that the Kerr-like medium switches the nonclassical squeezing periodically between the two quadratures with maximum values as those of the nondegenerate parametric amplifier (see the short-dashed and long-dashed curves in Fig. 3). This behavior is different from that of the two-mode squeezing for which nonclassical effects occur in one quadrature only. It is worth mentioning that the principal squeezing for the sum squeezing is typical as the solid curve in Figure 3, i.e. it is the envelope of the quadrature squeezing.

In conclusion, in this paper we give for the first time the competition between the down-conversion and Kerr-like processes from the point of view of generation nonclassical squeezing. We prove that generally the amount of the nonclassical squeezing obtained from each individual process is decreased as a result of their competition in the Kerr-down conversion system. This has been shown for different kinds of quadrature squeezing. We have also suggested more general form for the description of principal squeezing.

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